LESSON 5.1b

Solving Equations Using n^{th} Roots

Today you will:

- Understand when/why to only give the positive root as an answer (n^{th} root, n is even).
- Solve equations involving n^{th} roots.
- Practice using English to describe math processes and equations

Radical Form

Index

$$\sqrt[n]{a}$$

Radical Symbol

Base / Radicand

The “Square Root” Symbol is special!

No Index shown

$$\sqrt{a}$$

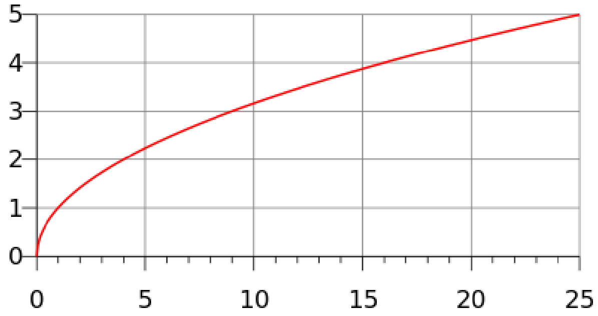
Square Root Symbol

Base / Radicand

Why in the textbook do they sometimes only show the positive answer when using $\sqrt{\ }$?

- Yes, all positive numbers have two square roots: the positive and negative ones.
- We call the positive one the principle root.
- Unless otherwise stated $\sqrt{\ }$ is defined to refer to the ***principle square root***.

- Examples of when you will only give the principle root:
  - $\sqrt{4}$ = 2 … "what is the square ***root*** (singular) of 4?"
  - $f(x) = \sqrt{x}$ … this is a function and must pass the vertical line test therefore must ignore negative answers.



- Examples of when you will give both the positive and negative answers:
  - $y = \sqrt{x}$ … this is an ***equation*** not a function and we are looking for all possible answers.
  - When directly asked "what are the square/nth ***roots*** (plural) of something."

Example of when to give both answers…

…plural…multiple roots…

UNDERSTANDING MATHEMATICAL TERMS

When n is even and $a > 0$, there are two real roots. The positive root is called the *principal root*.

Core Concept

Real nth Roots of a

Let n be an integer ($n > 1$) and let a be a real number.

n is an even integer.	**n is an odd integer.**
$a < 0$ No real nth roots	$a < 0$ One real nth root: $\sqrt[n]{a} = a^{1/n}$
$a = 0$ One real nth root: $\sqrt[n]{0} = 0$	$a = 0$ One real nth root: $\sqrt[n]{0} = 0$
$a > 0$ Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$	$a > 0$ One real nth root: $\sqrt[n]{a} = a^{1/n}$

Find the indicated real nth root(s) of a.

a. $n = 3$, $a = -216$

b. $n = 4$, $a = 81$

Example of when to give both answers...

...directly asked for multiple roots...

SOLUTION

a. Because $n = 3$ is odd and $a = -216 < 0$, -216 has one real cube root.

Because $(-6)^3 = -216$, you can write $\sqrt[3]{-216} = -6$ or $(-216)^{1/3} = -6$.

b. Because $n = 4$ is even and $a = 81 > 0$, 81 has two real fourth roots.

Because $3^4 = 81$ and $(-3)^4 = 81$, you can write $\pm\sqrt[4]{81} = \pm 3$ or $\pm 81^{1/4} = \pm 3$.

EXAMPLE 2 Evaluating Expressions with Rational Exponents

Evaluate each expression.

a. $16^{3/2}$

b. $32^{-3/5}$

Example of when to give just the positive answer…

…working with a constant…

SOLUTION

Rational Exponent Form	Radical Form
a. $16^{3/2} = (16^{1/2})^3 = 4^3 = 64$	$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$
b. $32^{-3/5} = \dfrac{1}{32^{3/5}} = \dfrac{1}{(32^{1/5})^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$	$32^{-3/5} = \dfrac{1}{32^{3/5}} = \dfrac{1}{(\sqrt[5]{32})^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

Find the real solution(s) of (a) $4x^5 = 128$ and (b) $(x - 3)^4 = 21$.

SOLUTION

a. $4x^5 = 128$ — Write original equation.

$x^5 = 32$ — Divide each side by 4.

$x = \sqrt[5]{32}$ — Take fifth root of each side.

$x = 2$ — Simplify.

The solution is $x = 2$.

b. $(x - 3)^4 = 21$ — Write original equation.

$x - 3 = \pm\sqrt[4]{21}$ — Take fourth root of each side.

$x = 3 \pm \sqrt[4]{21}$ — Add 3 to each side.

$x = 3 + \sqrt[4]{21}$ or $x = 3 - \sqrt[4]{21}$ — Write solutions separately.

$x \approx 5.14$ or $x \approx 0.86$ — Use a calculator.

The solutions are $x \approx 5.14$ and $x \approx 0.86$.

COMMON ERROR

When n is even and $a > 0$, be sure to consider both the positive and negative nth roots of a.

...we're solving an equation so looking for all possible answers!

A hospital purchases an ultrasound machine for \$50,000. The hospital expects the useful life of the machine to be 10 years, at which time its value will have depreciated to \$8000. The hospital uses the declining balances method for depreciation, so the annual depreciation rate r (in decimal form) is given by the formula

8000 10

$$r = 1 - \left(\frac{S}{C}\right)^{1/n}.$$

50,000

In the formula, n is the useful life of the item (in years), S is the salvage value (in dollars), and C is the original cost (in dollars). What annual depreciation rate did the hospital use?

SOLUTION

The useful life is 10 years, so $n = 10$. The machine depreciates to \$8000, so $S = 8000$. The original cost is \$50,000, so $C = 50{,}000$. So, the annual depreciation rate is

$$r = 1 - \left(\frac{S}{C}\right)^{1/n} = 1 - \left(\frac{8000}{50{,}000}\right)^{1/10} = 1 - \left(\frac{4}{25}\right)^{1/10} \approx 0.167.$$

The annual depreciation rate is about 0.167, or 16.7%.

Homework

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